# A SIMPLIFIED METHOD FOR THERMOELECTRIC HEAT PUMP OPTIMIZATION

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## Summary

A new, accurate, complete and easy-to-use method is introduced for present or potential users of thermoelectric heat pumps. This method replaces the typical performance graphs and tables which are either voluminous or specific for usually a room temperature heat sink.

The method was derived through computer analysis of the full temperature dependent thermoelectric theory applied to a generalized thermoelectric heat pump. The results are a few simple curves which allows one to optimize, select a cooler, or to determine how efficiently he is utilizing a cooler for literally any thermal/electrical condition between -125°C and +125°C.

## Introduction

This discussion will deal exclusively with single-stage thermoelectric heat pumps manufactured from thermoelectric materials produced by Marlow Industries. The results presented herein are accurate to less than 5% error over the temperature range of 150°K to 400°K (-123°C to +127°C). The results are similar to those described in a previous paper[1] on "standard commercial" materials.

## Design Parameter Description

### Current: $I\lambda$

Analysis of temperature dependent calculations of single-stage heat pump performance has lead to the discovery of systematic behavior of certain combinations of thermoelectric parameters at extreme conditions. The key was the definition of a combined variable: $I\lambda$; where I is the electrical current and $\lambda$ is the thermoelement length per unit cross-sectional area.

In the mathematical analysis of thermoelectric theory, the value of $I\lambda$ at either maximum efficiency or maximum cooling is independent of geometry. That is, $I\lambda$ is dependent only on boundary temperatures and material parameters which, in turn, are dependent on temperature.

### Voltage: V/N

The voltage, V, applied to a thermoelectric device, divided by the number of thermocouples in series, N, is also a very useful design parameter. The expanded mathematical expression for V/N involves only the quantity $I\lambda$ plus boundary temperatures. Consequently, the maximum cooling or maximum efficiency values are independent of all variables except boundary temperatures.

### Heat Pumping: $Q\lambda/N$

This parameter is a combination of $\lambda$ and N plus the heat pumped, Q, by the TE unit at the cold side. Similar to V/N, this parameter, $Q\lambda/N$, is a function only of $I\lambda$ and temperatures; and, therefore, is also invariant to all but boundary temperatures at the extreme conditions.

## Design Parameter Discussion

The usefulness of the three design parameters $I\lambda$, V/N and $Q\lambda/N$ will become quite apparent as one begins to operate with them. They provide direct access to device selection and design avoiding the "trial and error" iterative process normally encountered.

Once the hot side temperature, $T_H$, and cold side temperature, $T_C$, are defined, values for $I\lambda$, V/N and $Q\lambda/N$ are determinable for either the maximum cooling or maximum efficiency condition. Notice that the combination $Q\lambda/N \div (I\lambda \times V/N)$ reduces to Q/IV, the heat pumped at the cold side per watt of cooler input power. This value is exactly the Coefficient of Performance, COP, or "cooling efficiency." This allows feasibility judgements to be made prior to any further considerations for detailed device description. Many times this is the only information required such as is the design or selection of an appropriate heat sink.

The final cooler design consists simply of selecting values of Q, $\lambda$, N, I and V. Essentially, this reduces to the solution of five unknowns with three equations. That is, two variables can be arbitrarily selected subject to the constraints of the design parameter equations. For example, heat load, Q, and voltage, V, can be arbitrarily selected and $\lambda$, N, and I are subsequently calculated from the expressions for $I\lambda$, V/N and $Q\lambda/N$ completing the design. The detailed procedure for accomplishing this for any heat sink temperature from -125°C to +125°C is described in following sections.

## Maximum Parameters

The cooling performance of a thermoelectric device operating from a constant heat sink temperature is essentially parabolic. That is, $\Delta T = T_H - T_C$ increases rapidly at low currents but Joule heating eventually dominates at high currents resulting in the existence of a $\Delta T_m$ or maximum $\Delta T$. $I_m\lambda$ and $V_m/N$ are the respective design parameters at this condition. If heat is applied to the top of the cooler, the $\Delta T$ will decay linearly with applied heat. The intercept or point where $\Delta T = 0$ is $Q_m\lambda/N$.

As indicated earlier, these maximum values are functions of temperature only. In fact, they have discrete values given only the hot side temperature, $T_H$, as illustrated in Figure 1. For example, the design parameters for room temperature 300°K are: $I\lambda_m = 50.0$ amps/cm, $Q_m\lambda/N = 3.72$ watts/cm, $V_m/N = 0.114$ volts/cm, and $\Delta T_m = 73.2$°C. It should be emphasized that these values hold for any and all single-stage Marlow Industries coolers. If we set N to 10 couples and $\lambda$ to $10\text{cm}^{-1}$, the resulting device parameters are $I_m$ = 5.0 amps, $V_m$ = 1.14 volts and $Q_m$ = 3.72 watts.

### Design/Performance Chart

With the maximum parameters specified, the question remains as to the interrelationship of I, V, ΔT and Q over the continum from zero to their maximum value. Figure 2 illustrates this interdependence of the device performance parameters normalized to their respective maximum values. Notice that the locus of points for the "optimum" combination of I, ΔT and Q is also shown.

### Design/Selection Process

The process for utilizing Figures 1 and 2 for cooler design and selection is illustrated in Figure 3. Each "box" contains the step number in the process, the source for the data, the variable to be defined and a space provided for entry of that value. Once the value for N/λ is determined, a heat pump can be selected from Table 1.

TABLE 1 - STANDARD HEAT PUMPS

| Model No. | N | λ ($cm^{-1}$) | N/λ (cm) |
|---|---|---|---|
| MI 1020 | 7 | 25.20 | 0.278 |
| MI 1021 | 17 | 25.20 | 0.675 |
| MI 1022 | 31 | 25.20 | 1.230 |
| MI 1023 | 71 | 25.20 | 2.817 |
| MI 1050 | 35 | 9.21 | 3.800 |
| MI 1060 | 7 | 7.27 | 0.963 |
| MI 1061 | 17 | 7.27 | 2.338 |
| MI 1062 | 31 | 7.27 | 4.264 |
| MI 1063 | 71 | 7.27 | 9.766 |
| MI 1092 | 31 | 5.23 | 5.927 |
| MI 1120 | 31 | 4.14 | 7.488 |
| MI 1142 | 31 | 3.27 | 9.480 |

The process can be best illustrated by an example. Let us consider the selection of a standard cooler for the following conditions:

$T_H$ = 350°K
$T_C$ = 290°K
Q = 10 watts

The values obtained in performing each step are listed in Table 2.

Each step in this process is straight-forward up to steps #14 & 15 where an actual device is selected. The selection basis was to pick one from the table in the N/λ range specified by the values in steps #12 & 13 favoring, of course, the optimum or step #12 value.

Truly optimum coolers can be designed by: (a) selecting one closest to the optimum N/λ and (b) determine the "custom" λ holding N constant. In practice, this amounts to a relatively inexpensive modification of adjusting the TE pellet size. This optimum case also provides "upper bound" performance so that cost/performance trade-offs can be considered.

TABLE 2
DESIGN/SELECTION SUMMARY

| Step | Source | Parameter | Value |
|---|---|---|---|
| 1 | Input | $T_H$ (°K)...................... | 350 |
| 2 | Figure #1 | $Q_m\lambda/N$ (watts/cm)............ | 4.5 |
| 3 | Figure #1 | $\Delta T_m$(°C)..................... | 95 |
| 4 | Input | $T_C$ (°K)...................... | 290 |
| 5 | Calculation | ΔT (°C)...................... | 60 |
| 6 | Calculation | $\Delta T/\Delta T_m$..................... | 0.63 |
| 7 | Figure #2 | Optimum $Q/Q_m$.............. | 0.18 |
| 8 | Figure #2 | Maximum $Q/Q_m$.............. | 0.36 |
| 9 | Calculation | Optimum Qλ/N (watts/cm)..... | 0.81 |
| 10 | Calculation | Maximum Qλ/N (watts/cm)..... | 1.62 |
| 11 | Input | Q (watts).................. | 10 |
| 12 | Calculation | Optimum N/λ(cm)............ | 12.34 |
| 13 | Calculation | Maximum N/λ(cm)............ | 6.17 |
| 14 | Table 1 | N.......................... | 71 |
| 15 | Table 1 | λ($cm^{-1}$)..................... | 7.27 |

Device Selected MI 1063

### Performance/Analysis Procedure

Once a thermoelectric cooler has been selected and N and λ are defined, the performance under any conceivable cooling condition can be either forecasted or analyzed using the same charts as for design. This is accomplished by following the step-by-step procedure illustrated in Figure 4. A summary of final details of the example case is given in Table 3.

TABLE 3
Performance/Analysis Summary

| Step | Source | Parameter | Value |
|---|---|---|---|
| 1 | Input | $T_H$ (°K)...................... | 350 |
| 2 | Figure #1 | $\Delta T_m$(°C)..................... | 95 |
| 3 | Figure #1 | $Q_m\lambda/N$ (watts/cm)............ | 4.5 |
| 4 | Figure #1 | $I_m\lambda$ (amps/cm)............... | 49 |
| 5 | Figure #1 | $V_m/N$ (volts)................ | 0.143 |
| 6 | Input | N.......................... | 71 |
| 7 | Input | λ ($cm^{-1}$).................... | 7.27 |
| 8 | Calculation | $Q_m$ (watts)................ | 43.9 |
| 9 | Calculation | $I_m$ (amps).................. | 6.74 |
| 10 | Calculation | $V_m$ (volts)................. | 10.2 |
| 11 | Input | ΔT (°C)..................... | 60 |
| 12 | Calculation | $\Delta T/\Delta T_m$.................... | .63 |
| 13 | Input | Q (watts).................. | 10 |
| 14 | Calculation | $Q/Q_m$...................... | .228 |
| 15 | Figure #2 | $I/I_m$...................... | .62 |
| 16 | Calculation | I (amps)................... | 4.18 |
| 17 | Figure #2 | $V/V_m$...................... | .64 |
| 18 | Calculation | V (volts).................. | 6.53 |

Notice that at the completion of step #10, Figure 2 is converted to a performance chart for this specific cooler for the specified $T_H$. The remaining steps are to merely define the specific operating conditions to produce the cooling performance specified in the example. Actually, any two parameters can be selected arbitrarily per the note in Figure 4.

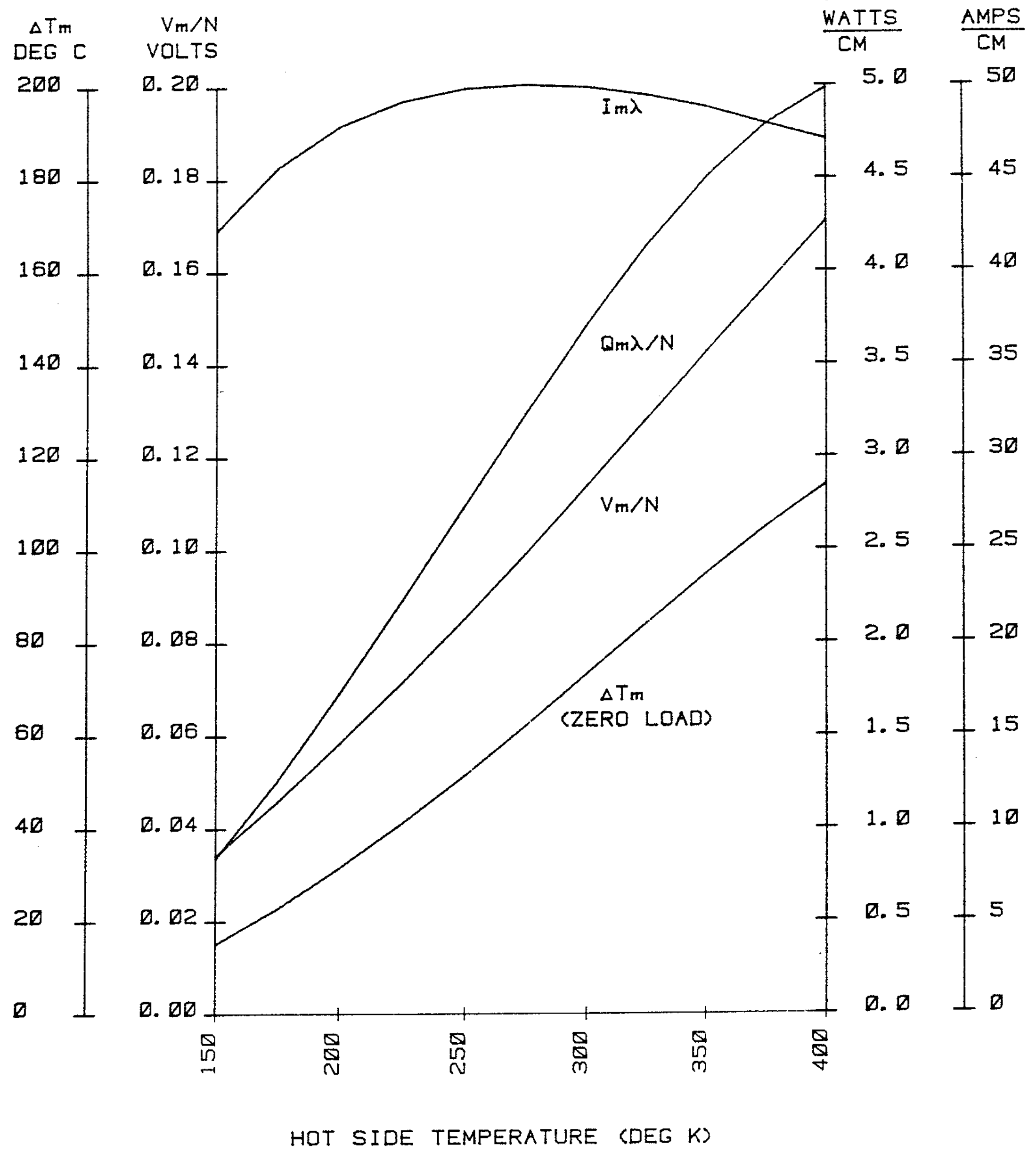
## Conclusions

Data has been presented in two figures which can be used to design, select, optimize, characterize and analyze any single-stage Marlow Industries thermoelectric heat pump for use in any conceivable cooling application within a temperature range of -123°C to +127°C. This data has been generated through the recognition of three "design parameters" from thermoelectric theory and empirically derived normalized parameter interdependence. The result is an accurate, complete and easy-to-use system for those using single-stage thermoelectric heat pumps.

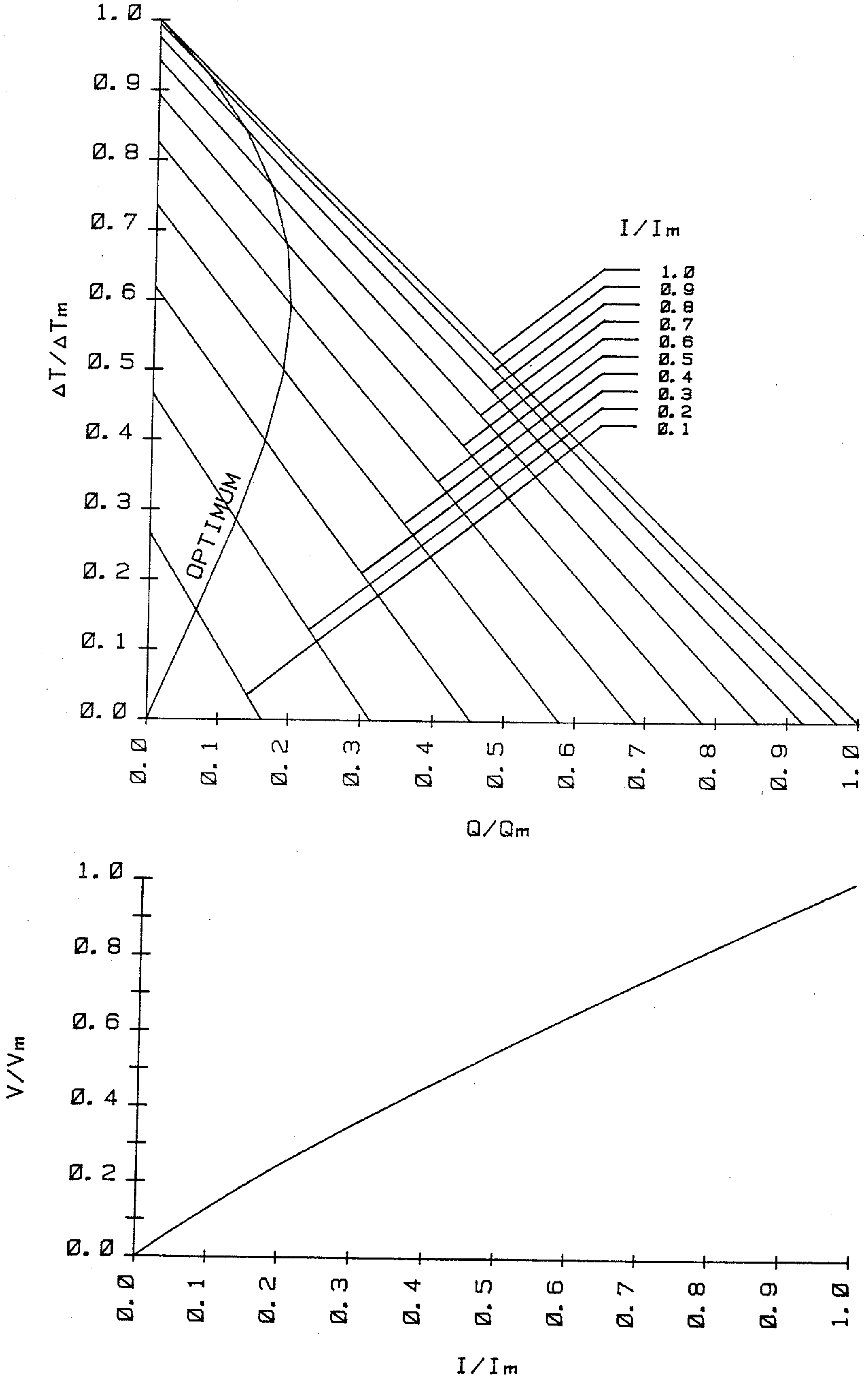
## Reference

[1] R. J. Buist, Proc. of the 14th IECEC (1979)

MAXIMUM DESIGN PARAMETERS VS. HOT SIDE TEMPERATURE FIGURE 1

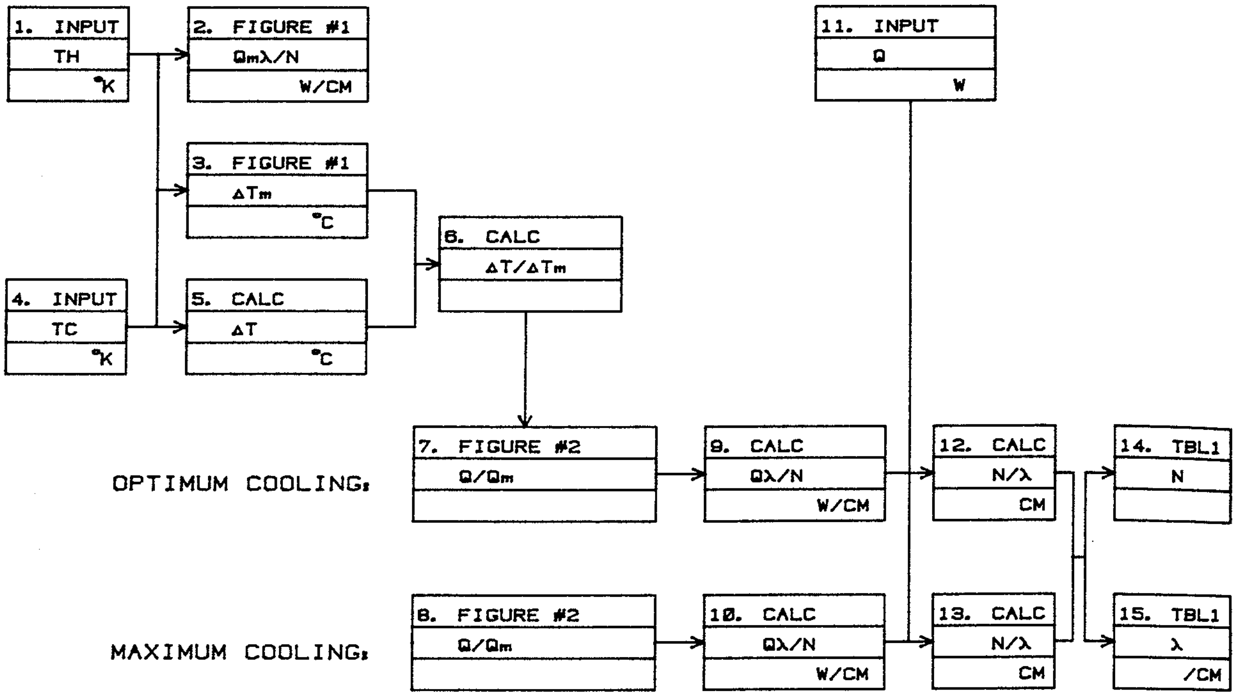


DESIGN/PERFORMANCE CHART FIGURE 2



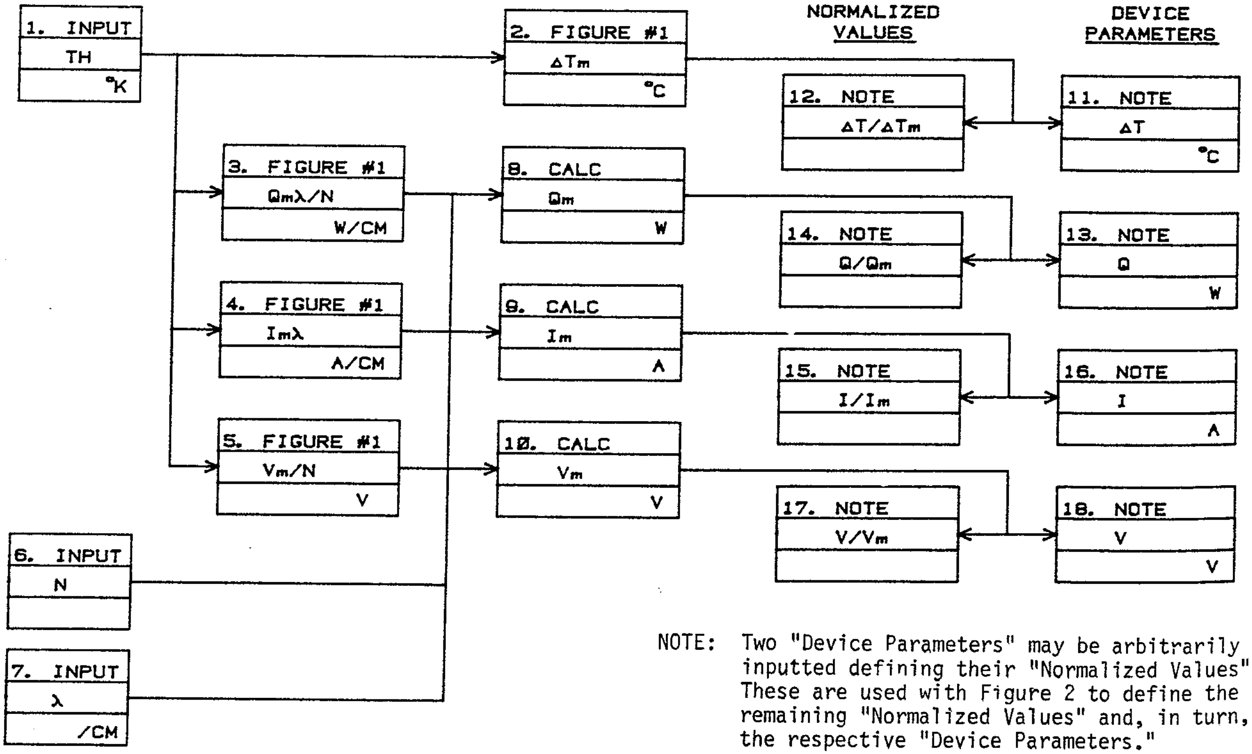
## DESIGN/SELECTION PROCEDURE

FIGURE 3



## PERFORMANCE/ANALYSIS PROCEDURE

FIGURE 4



NOTE: Two "Device Parameters" may be arbitrarily inputted defining their "Normalized Values". These are used with Figure 2 to define the remaining "Normalized Values" and, in turn, the respective "Device Parameters."