# Calculation of Thermoelectric Power Generation Performance Using Finite Element Analysis

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**Abstract**

Recent papers [1], [2] have covered the merits of using finite elements to calculate the thermoelectric device performance for steady-state conditions. Likewise, papers [3], [4] have covered the use of finite elements to model transient cooling conditions. It remains then to model power generation performance with finite elements and compare that model with various other modeling techniques.

Analysis was based on a single pellet for simplification. The pellet was modeled by treating the differential equations as closed form, finite equations across a small section of the pellet. The many sections, or finite elements, comprised the total pellet. Temperature dependent properties were incorporated into the model.

The finite element analysis predicted different results than the temperature-averaging techniques as was to be expected. Finite element analysis should be used when critical optimization is required since it is able to determine accurately the nature of the thermoelectric effects of materials whose properties are highly temperature dependent. Averaging schemes, by their very nature, lose modeling information and are less accurate.

**Introduction**

There are several ways to calculate the power generation performance of a thermoelectric couple, either by averaging schemes or by the use of finite elements. The advantage of the averaging schemes is that an immediate answer is obtained from simplified analytical equations. Finite elements require much iteration and, therefore, more time to obtain results. However, ever higher speed computers make this time difference increasingly negligible. The advantages of using finite elements over averaging techniques is accuracy and true optimizations. Prototype and experiment costs will surely be reduced when the model agrees precisely with experiment. Also, one can be certain that the higher performance will not be achieved by any other design.

The object of this study was to compare two models which used averaging techniques and a model which used finite element analysis. Graphs are presented to illustrate comparisons. For the purposes of this paper only one - dimensional analysis was investigated. Also, such effects as contact resistances were not included in the analysis since only a basic comparison of calculation methods was desired. Furthermore, the models did not include effects of passive heat losses from radiation and air conduction and convection. Such effects could readily be incorporated though to provide a model that would more closely match reality.

**Computational Models**

All three computational models are based on the following equations [5]:

$$Q_H = \alpha I T_H - \tfrac{1}{2} I^2 R + K \Delta T \qquad (1)$$

$$P = I^2 R_L \qquad (2)$$

$$I = V_{oc} / (R + R_L) \qquad (3)$$

$$\Delta T = T_H - T_C \qquad (4)$$

$$V_{oc} = \alpha \Delta T \qquad (5)$$

$$R = \rho L / A \qquad (6)$$

$$K = \kappa A / L \qquad (7)$$

$$Q_H = P + Q_C \qquad (8)$$

$$Z = \alpha^2 / \rho / \kappa \qquad (9)$$

$Q_H$ = heat input to the thermoelectric generator
$\alpha$ = Seebeck coefficient
$\kappa$ = thermal conductivity
$T_H$ = hot side temperature
$P$ = output power
$V_{OC}$ = open circuit voltage
$L$ = length of pellet
$Q_C$ = waste heat
$Z$ = figure-of-merit
$\rho$ = electrical resistivity
$I$ = current
$T_C$ = cold side temperature
$R_L$ = load resistance
$A$ = cross sectional area of pellet

Figure 1 shows a typical, schematic representation of a thermoelectric generator.

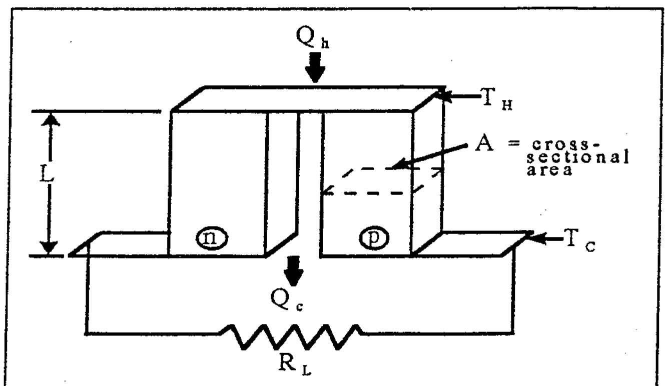


Figure 1. Thermoelectric Generator

The material properties of the thermoelectric material used for this analysis were obtained from an actual test result on a module as measured with TE Technology, Inc.'s TS-205. The temperature dependent properties were then extrapolated at the extreme temperatures where test data was not available yet.

Model 1: $P(T_{HC})$
This model used material properties based on the average of $T_H$ and $T_C$ of the pellet.

Model 2: $P_{avg}$
This model used an integral average of the material properties. For example the average Seebeck coefficient was given by:

$$\bar{\alpha} = \frac{1}{T_H - T_C}\int_{T_C}^{T_H} \alpha(T)dT$$

Model 3: $P_{fea}$
This model splits the pellet into many small sub-sections, or elements. Eq. (1) was applied to each element whereby the heat leaving one element became the heat entering the subsequent element. The total power output was the sum of the power outputs of each element, as calculated from Eq. (8). The material properties were implicitly determined based on the temperature distribution calculated by a given heat flow. The heat flow, $Q_H$, into the pellet was parametrically altered by numerous iterations until the desired hot side temperature was obtained, assuming a given cold side temperature. Figure 2 shows a schematic of the $P_{fea}$ model.

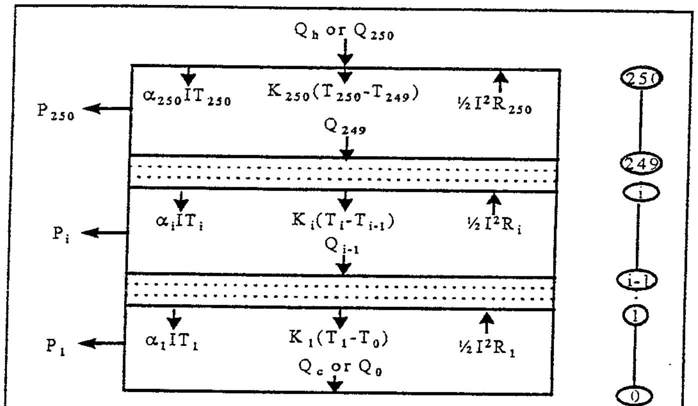


Figure 2. $P_{fea}$ Model Schematic

**Results**

The determination of maximum efficiency was calculated by setting $R_L = (1+ZT_{HC})^{1/2}R$ [5] where $Z$ was calculated at $T_{HC}$ for $P(T_{HC})$; $T_{HC} = (T_H + T_C)/2$. For $P_{avg}$, $R_L$ was calculated in the same manner except that the $Z$ was calculated from the integral-averaged properties.

$P_{fea}$ used a slightly different approach. The optimum $R_L$ was not the true optimum when determined in the same manner as for $P(T_{HC})$ and $P_{avg}$. Rather the optimum was calculated from $(R_{L1}+R_{L2})/2$. $R_{L1}$ was given by $(1+ZT_{avg})^{1/2}R$ where $T_{avg} = \left(\sum_{i=1}^{n} T_i\right)/n$. The $Z$ was calculated at $T_{avg}$. $R_{L2}$ was determined by $\left[\sum_{i=1}^{n}(1+Z_i(T_i+T_{i+1})/2)^{1/2}\right]\frac{R}{n}$.

However, there were only slight differences in the optimum $R_L$ determined by any of the above means. The table below shows efficiency results from $P_{fea}$ according to different methods used in calculating the optimum $R_L$.

The methods are identified in Table 1 as follows:
① $R_L = (1+ZT_{HC})^{1/2}R$
② $R_{L1} = (1+ZT_{avg})^{1/2}R$
③ $R_{L2} = \left[\sum_{i=1}^{n}(1+Z_i(T_i+T_{i+1})/2)^{1/2}\right]\frac{R}{n}$
④ $(R_{L1}+R_{L2})/2$

Table 1: Efficiency Based on Calculation of $R_L$

| Method | efficiency, % $T_H$ = 470K $T_C$ = 300K | efficiency, % $T_H$ = 600K $T_C$ = 300K |
|---|---|---|
| ① | 5.608627 | 5.723894 |
| ② | 5.608843 | 5.727157 |
| ③ | 5.608812 | 5.727169 |
| ④ | 5.608895 | 5.727174 |

At a $\Delta T$ = 170 K, the percent difference between method ① and ④ was only 0.005%. However, at a $\Delta T$ = 300 K, the percent difference between method ① and ④ was 5.73%. The averaging scheme represented by method ① is inadequate, especially at large $\Delta T$'s.

Figure 3 shows the maximum efficiency as determined by each model at different $T_H$'s and at a constant $T_C$ = 300K. For small temperature differences, the three models provided identical results. However, $P_{fea}$ yielded a significantly lower efficiency at the higher temperature differences.

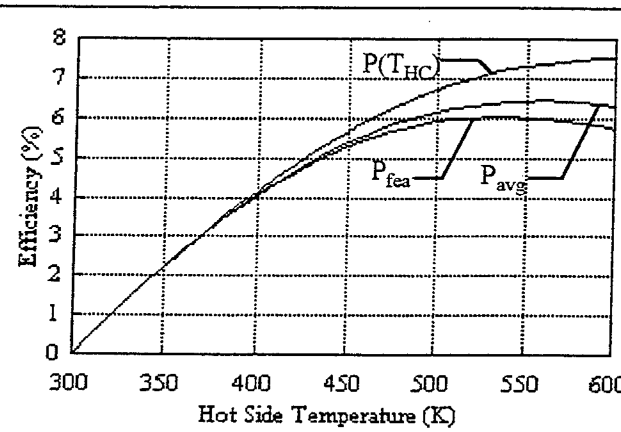


Figure 3. Maximum Efficiency ($T_C$ = 300 K)

Figure 4 shows the maximum power as determined by each model at different $T_H$'s and at a constant $T_C$ = 300K. The maximum power was calculated by setting $R_L = R$. The models were based on 127 couples of 1.4 x 1.4 x 1.15mm pellet dimensions. Again at the smaller temperature differences, the $P(T_{HC})$ and $P_{avg}$ provided sufficiently accurate answers when compared with $P_{fea}$. At the higher temperature differences, the variation was not quite as dramatic as for maximum efficiency. Nonetheless, the $P_{fea}$ yielded a lower power output than calculated by $P(T_{HC})$ or $P_{avg}$.

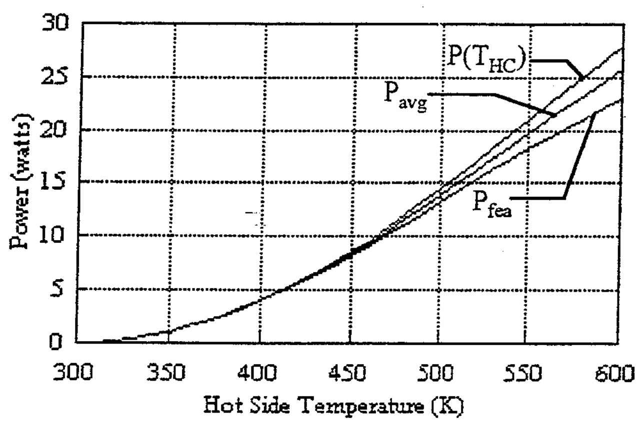


Figure 4. Maximum Power ($T_C$ = 300 K)

Figures 5,6,7,8 and 9 show how current, load voltage, heat input and efficiency and output power compare according to the three models as a function of load resistance, keeping $\Delta T$ constant. Clearly, $P_{avg}$ is a better model than $P(T_{HC})$. In Figure 5, $P_{avg}$ and $P_{fea}$ calculated essentially the same load voltage, indicating that $P_{avg}$ is a superior model to $P(T_{HC})$. As shown in Figure 6, both $P(T_{HC})$ and $P_{avg}$ over-predict the current. Figure 7 shows clearly that $P(T_{HC})$ was just too simplistic a model to provide sufficient accuracy.

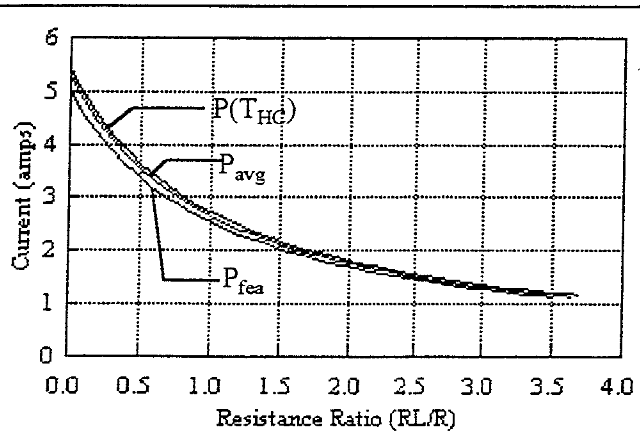


Figure 5. Effect of Load Resistance on Current
($T_H$ = 500 K, $T_C$ = 300 K)

Figure 8 does show that the optimum resistance ratio for maximum efficiency was the same regardless of the model employed to determine it. However, the resistance ratios were not *exactly* the same. Table 2 shows how the ratios compared for the three models. While the optimum resistance ratios were essentially the same, the $P_{fea}$ calculated significantly lower efficiencies than the other two models.

Table 2: Resistance Ratio for Optimum Efficiency

| Model | $R_L/R$ |
|---|---|
| $P(T_{HC})$ | 1.323393 |
| $P_{avg}$ | 1.291606 |
| $P_{fea}$ | 1.302489 |

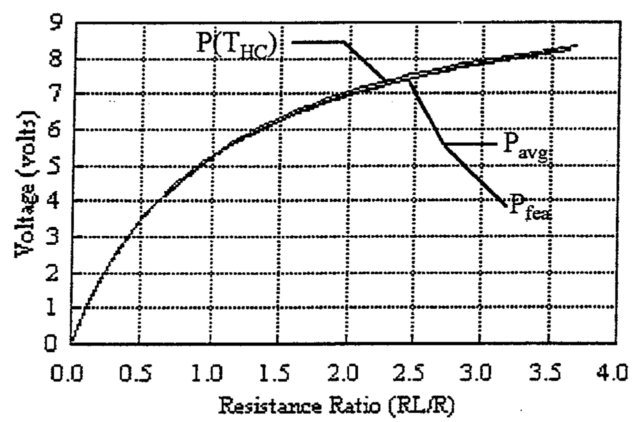


Figure 6. Effect of Load Resistance on Load Voltage
($T_H$ = 500 K, $T_C$ = 300 K)

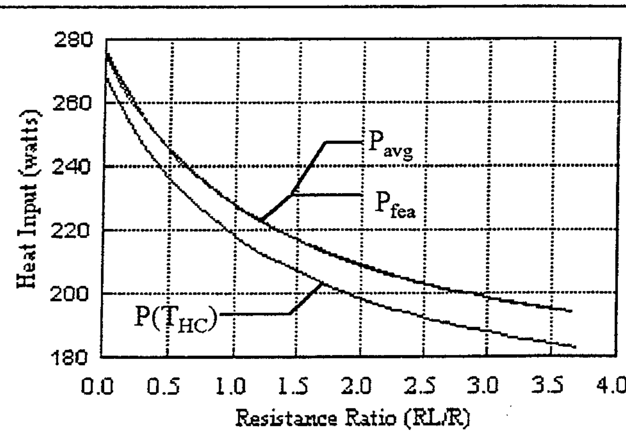


Figure 7. Effect of Load Resistance on Heat Input
($T_H$ = 500 K, $T_C$ = 300 K)

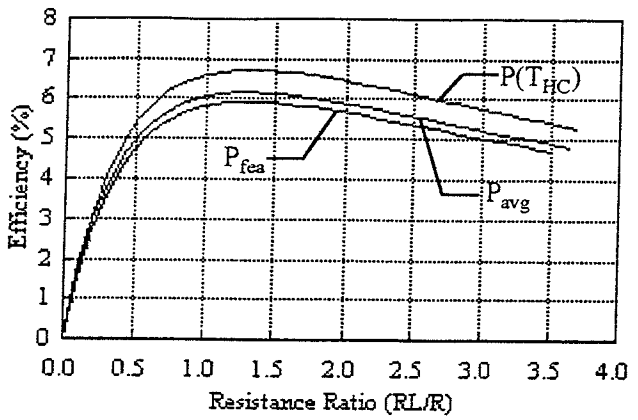


Figure 8. Effect of Load Resistance on Efficiency ($T_H = 500$ K, $T_C = 300$ K)

Figure 9 indicates that, regardless of the model, the resistance ratio for maximum power was unity, according to what the closed form, analytical equations predict. However, this may not be precisely true for all types of materials in all situations. As evidenced in Table 2, the closed form equations, do not necessarily predict the true optimum. Nonetheless, $P_{fea}$ calculated a lower power level than the other two models.

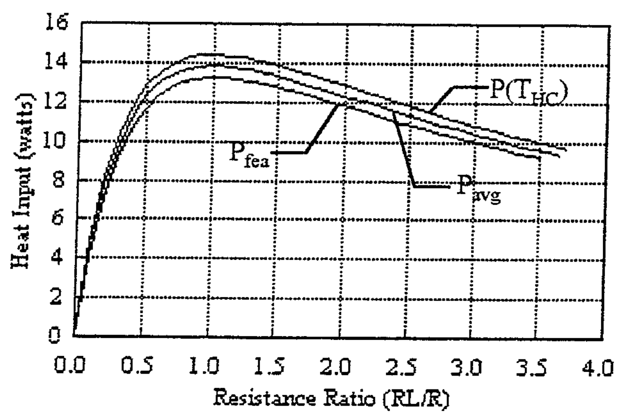


Figure 9. Effect of Load Resistance on Output Power ($T_H = 500$ K, $T_C = 300$ K)

**Computation Time and Accuracy**

The $P_{fea}$, using 250 elements in the model, required about 40 seconds to arrive at a solution when performed on a 90 MHz clock speed CPU. $P(T_{HC})$ and $P_{avg}$ provided practically instantaneous results. When the temperature differences are large this additional computation time is probably justifiable. However, $P_{fea}$ is itself only as accurate as the number of elements used in the model. The more elements that are used, the greater will be the internal accuracy, but also the greater will be the computation time. Table 3 shows comparisons of the $P_{fea}$ model using 250 elements and 251 elements for the calculation of maximum efficiency at $T_H$ = 600 K and $T_C$ = 300 K, 127 couples, 1.4 x 1.4 x 1.5 mm pellet dimensions.

Table 3: $P_{fea}$ Internal Accuracy Comparison

| # of elements | Load Voltage V | Current A | Heat Input W | $R_L/R$ |
|---|---|---|---|---|
| 250 | 8.627417 | 2.652974 | 399.644 | 1.22133 |
| 251 | 8.626449 | 2.653348 | 399.651 | 1.22103 |
| % change | 0.011 | 0.014 | 0.0018 | 0.025 |

The use of 250 elements appears to have been sufficient for accurately calculating performance.

**Conclusions**

The $P_{fea}$ has been shown to yield markedly different results when compared with $P(T_{HC})$ and $P_{avg}$ results. The difference became more significant as the $\Delta T$ increased. There were also slight variations between the models for the determined optimum resistance ratio for maximum efficiency. While reasonable accuracy for determining the optimum resistance ratio for maximum efficiency could be achieved by using any of the three models, the variations could be much more exaggerated if the temperature dependent thermoelectric material properties were highly non-linear. Only the $P_{fea}$ is sophisticated enough to correctly determine the true optimum. $P_{fea}$ could be made even more sophisticated by including radiation, convection and conduction passive heat losses. These loss calculations could be easily included on an element-by-element basis whereas it would be quite difficult to correctly include in $P(T_{HC})$ or $P_{avg}$. Thus, the $P_{fea}$ yields the best accuracy, it can easily model complex, real world situations and it calculates the true optimum.

**References**

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